Electoral competition with disagreement

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Abstract. What happens when candidates disagree about the preferences of voters in a spatial model of electoral competition? Even a small amount of disagreement allows for a large policy gap between the candidates in equilibrium. This is studied for office-motivated candidates and policy-motivated candidates.

1. Introduction and model

In standard spatial models of electoral competition following Hotelling (1929), Black (1948), and Downs (1957), it is assumed that the candidates have the same beliefs about the policy preferences of the voters. This paper shows that the equilibrium of these models is particularly sensitive to this assumption.

In the model considered in this paper, there are two candidates $i \in \{A, B\}$ who simultaneously announce a policy position $\rho_i \in [0, 1]$. The unit interval $[0, 1]$ is the domain of the possible policies: if $x < y$ are two policy positions, then x is more liberal than y, and equivalently y is more conservative than x.

There is a set of voters. As in Downs (1957), each voter j has an ideal policy $\theta_j \in [0, 1]$ and casts its vote for the candidate with a policy position closest to θ_j. If the candidates are equally close to θ_j, then voter j randomly casts its vote with equal probability among the candidates. As standard in this literature, this behavior can be microfounded when voters have single-peaked preferences.

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The candidate receiving the greatest fraction of the vote wins the election. If both candidates receive the same fraction of votes, then the election is decided by randomly choosing a candidate with equal probability.

Candidates are the strategic decision makers, corresponding to the announcement of policy positions. The paper considers two different utility functions for the candidates. In the first setup in this paper in Section 2, candidates are office-motivated, so they only care about winning the election. In this case, each candidate chooses a policy to maximize its probability of winning the election. As standard in this literature, this can be microfounded when candidates get utility 1 from winning and utility 0 from losing. In the second setup in this paper in Section 3, candidates are policy-motivated, so they care about the policy that is enacted, according to their utility function over policies. In this case, each candidate chooses a policy to maximize its expected utility over the policy that is enacted.

In either setup, the decision problem facing the candidates involves their beliefs about the fraction of the vote they will get for each possible arrangement of policy positions. So far, this follows the standard spatial model of electoral competition.

The key departure from the standard model is that candidates do not necessarily share the same beliefs about the voters. Specifically, candidate $i \in \{A, B\}$ believes that it would receive a fraction $F_i(\rho)$ of the vote if each voter with an ideal policy position at least as liberal as ρ voted for candidate i. Equivalently, $F_i(\rho)$ is the fraction of the voters that candidate i believes to have an ideal policy that is at least as liberal as the policy position ρ. If the actual distribution of voters' ideal policies is F, then the beliefs of the candidates are correct if $F_A = F_B = F$. This implies, in particular, that the standard setup of a spatial model of electoral competition is a special case of the setup in this paper. The distributions F_A and F_B are assumed atomless with everywhere strictly positive density.

Thus, as a key point of departure from the literature, candidates are allowed to have different beliefs about the preferences of the voters, when $F_A \neq F_B$. Furthermore, incorrect beliefs are allowed. In particular, when candidates have different beliefs, at least one candidate must have incorrect beliefs. There is considerable empirical evidence that different people have different and often incorrect political beliefs, in particular but not only about the political views of their political opponents (e.g., Kuklinski et al. (2000), Levendusky and Malhotra (2016), Ahler and Sood (2018), and Jerit and Zhao (2020)). This paper takes beliefs as primitives, and beliefs do not need to satisfy any (mutual) consistency conditions. In particular, this paper does not require that candidates share a "common prior." Some other models allow for candidates to have private information about the voters, maintaining the common prior assumption. As discussed further in Section 4, the predictions from those models differ substantially from the prediction of the models considered in this paper. Morris (1995) discusses arguments for studying models without a common prior. In particular, Aumann (1976) implies (under suitable conditions) that a common prior should lead to shared beliefs; given the empirical evidence cited above about political beliefs,[1] this would seem to suggest that the common prior assumption may be worth relaxing in settings like this.[2]

We say that the candidates "disagree about the median voter" if $F_A^{-1}(\frac{1}{2}) \neq F_B^{-1}(\frac{1}{2})$. This is a very weak notion of disagreement, since it only requires that the candidates disagree about the preferences of the median voter. Moreover, it does not require any particular *magnitude* of disagreement. Conversely, if $F_A^{-1}(\frac{1}{2}) = F_B^{-1}(\frac{1}{2})$, then we say that the candidates "agree on

[1]This cited empirical literature is indeed about "beliefs"; this literature does not use "political beliefs" as a synonym for "political preferences," as sometimes happens in other places.

[2]Another motivation for the possibility that candidates have different beliefs comes from the literature on partial identification (e.g., Tamer (2010), Bontemps and Magnac (2017), Ho and Rosen (2017), Molinari (2020), Kline, Pakes, and Tamer (2021), and Kline and Tamer (2023)). For a stylized connection, introduce a binary random variable R that divides the overall voting population into two subpopulations. The subpopulation with $R = 1$ are those voters who respond to a political survey; the subpopulation with $R = 0$ are those voters who do not respond. Then, following the same reasoning as in Manski (2007, Section 2.1), by the law of total probability, the true distribution of voters' ideal policies is $F(\rho) = F(\rho|R = 1)P(R = 1)+F(\rho|R = 0)P(R = 0)$. The political survey can be taken to provide the value of $F(\rho|R = 1)$. But such data says nothing about $F(\rho|R = 0)$. Instead, candidates must "fill in" $F(\rho|R = 0)$ using their own totally subjective beliefs, which cannot be learned from such data. When candidate i believes non-respondents have ideal policies given by the subjective distribution $F_i(\rho|R = 0)$, then candidates can have different overall beliefs about the distribution of voters' ideal policies, even when there is such objective data about *some* of the voters. This can be formalized to persist in a Bayesian estimation setup, as in Poirier (1998), Moon and Schorfheide (2012), Kline and Tamer (2016), and Giacomini and Kitagawa (2021). See also for example Dominitz and Manski (2026) for a direct analysis of such ideas in political polling. Note that this is only a motivation for allowing candidates to have different beliefs, without further application to the analysis in this paper. Therefore, non-interested readers may totally skip this footnote without losing the main thread of the paper.

the median voter." The paper shows that the predictions of the model depend sharply on whether the candidates "agree" or "disagree" about the median voter.

The paper characterizes the set of equilibria in this model, both with office-motivated candidates and policy-motivated candidates, and shows that the set of equilibria when the candidates "disagree about the median voter" is very different from the "median voter theorem" that arises in the standard spatial model of electoral competition when the candidates share the same beliefs about the voters.

The first setup in this paper concerns office-motivated candidates. If the candidates actually "agree about the median voter", then the equilibrium exhibits the "median voter theorem" as expected. This replicates the equilibrium in the standard model, where both candidates announce the ideal policy position of the "median voter".

However, the paper shows that even a *small* amount of "disagreement about the median voter" allows for a *large* policy gap between the candidates in equilibrium. The distance between the policy positions announced by the candidates can be very large if they disagree even minimally about the preferences of the voters. For example, Corollary 2.3 considers a case where the candidates disagree about the median voter by an arbitrarily small amount, and shows that there is an equilibrium where the candidates announce policies on the extreme opposite sides of the policy space. Moreover, as illustrated by this result, in equilibrium candidates do not necessarily announce the policy position that is ideal for the voter they believe to be the median voter. This policy announcement of the candidates is *an* equilibrium, but the equilibrium is not unique.

The paper shows related results when candidates are policy-motivated. The set of equilibria depends on the arrangement of the candidates' ideal policies and the candidates' beliefs about the voters. For example, Corollary 3.4 considers a case where each candidate believes that the median voter "agrees" with but is more moderate than the candidate itself, and shows that it can be an equilibrium for both candidates to announce their ideal policies. This can happen even if there is only a *small* amount of "disagreement about the median voter," including cases where the candidates themselves have substantially different ideal policies.

Overall, the results in this paper show that *even a small amount of* disagreement between the candidates about the preferences of the voters *can* generate a *large* policy gap between the candidates in equilibrium. This implies that the "median voter theorem" is particularly sensitive to the condition that the candidates share beliefs about the preferences of the voters. As discussed more in Section 4, this high sensitivity stands in contrast to other results in the literature on spatial voting models, where the "median voter theorem" is often found to be relatively robust to (small) deviations from the standard spatial voting model setup. This existing literature includes other results that also focus on what the candidates believe; therefore, whereas some existing results indicate that the "median voter theorem" is robust to *certain* deviations from candidates sharing the same beliefs about the voters, this paper shows that the "median voter theorem" is not robust to this particular deviation from candidates sharing the same beliefs about the voters.

Section 2 studies office-motivated candidates, Section 3 studies policy-motivated candidates, and Section 4 contrasts the results to the literature. Appendix A provides the proofs.

2. Equilibrium with office-motivated candidates

In this section, candidates are office-motivated; a candidate's utility is 1 upon winning the election and 0 upon losing the election. Consequently, each candidate is trying to maximize its probability that it wins the election.

Both candidates know that voters cast their votes based on their policy preferences, and the candidates' policy announcements ρ_A and ρ_B. Similar to Downs (1957), candidate i believes it will win (lose) if it believes it will get strictly more (less) than $\frac{1}{2}$ of the vote, according to its beliefs about the voters F_i. Unlike Downs (1957), possibly $F_A \neq F_B$. Also, candidate i believes it will tie the vote (and hence actually win office with probability $\frac{1}{2}$) if it believes it will get exactly $\frac{1}{2}$ of the vote, according to its beliefs about the voters F_i.

Theorem 2.2 completely characterizes pure strategy equilibrium arrangements of policy announcements. However, first, Lemma 2.1 provides a useful (but obvious) characterization of when the candidates "disagree about the median voter" (i.e., $F_A^{-1}(\frac{1}{2}) \neq F_B^{-1}(\frac{1}{2})$).

Lemma 2.1. *The candidates "disagree about the median voter" (i.e., $F_A^{-1}(\frac{1}{2}) \neq F_B^{-1}(\frac{1}{2})$) if and only if either: $F_A(\rho) < \frac{1}{2}$ and $F_B(\rho) > \frac{1}{2}$ for some $\rho \in [0,1]$, or $F_A(\rho) > \frac{1}{2}$ and $F_B(\rho) < \frac{1}{2}$ for some $\rho \in [0,1]$.*

Theorem 2.2. *Suppose candidates are office-motivated. Then a pure strategy equilibrium exists, and the only pure strategy equilibria are:*

(2.2.1) $\rho_A < \rho_B$ *that satisfy* $F_A(\frac{\rho_A+\rho_B}{2}) > \frac{1}{2}$ *and* $F_B(\frac{\rho_A+\rho_B}{2}) < \frac{1}{2}$

(2.2.2) $\rho_A > \rho_B$ *that satisfy* $F_A(\frac{\rho_A+\rho_B}{2}) < \frac{1}{2}$ *and* $F_B(\frac{\rho_A+\rho_B}{2}) > \frac{1}{2}$

(2.2.3) $\rho_A = \rho = \rho_B$ *when* $F_A(\rho) = \frac{1}{2} = F_B(\rho)$

Equilibria 2.2.1 and 2.2.2 happen only when the candidates disagree about the median voter; Equilibrium 2.2.3 happens only when the candidates agree about the median voter.

And all equilibria for a given F_A and F_B have the same ordering of policy announcements, defined as:

(1) $\rho_A < \rho_B$*; all of which have* $F_A(\frac{\rho_A+\rho_B}{2}) > \frac{1}{2}$ *and* $F_B(\frac{\rho_A+\rho_B}{2}) < \frac{1}{2}$

(2) $\rho_A > \rho_B$*; all of which have* $F_A(\frac{\rho_A+\rho_B}{2}) < \frac{1}{2}$ *and* $F_B(\frac{\rho_A+\rho_B}{2}) > \frac{1}{2}$

(3) $\rho_A = \rho_B = \rho$*; which has* $F_A(\rho) = \frac{1}{2}$ *and* $F_B(\rho) = \frac{1}{2}$

According to Theorem 2.2, candidates announce the same policy position exactly in case they agree about the median voter, as in Equilibrium 2.2.3. And, in that case, both candidates announce the same policy position as the ideal policy of the median voter. Thus, when candidates *do* agree about the median voter, Theorem 2.2 reproduces the standard "median voter theorem."

However, Theorem 2.2 also shows that if the candidates disagree about the median voter, the candidates *must* make different policy announcements in equilibrium. This result shows that disagreement about voter preferences can be a simple explanation for real-world candidates making different policy announcements.

Therefore, Theorem 2.2 also shows that the "median voter theorem" arises *only* when candidates agree about the median voter. In particular, note that the candidates *do not* necessarily announce the policy position that is ideal for the voter they believe to be the median voter. This represents a *qualitative* difference from the standard "median voter theorem." In fact, even a small amount of disagreement about the median voter opens up the possibility for candidates to make significantly different policy announcements. The magnitude of the policy gap is explored more, below.

Rather than the "median voter theorem" holding, when candidates disagree about the median voter, as in Equilibria 2.2.1 and 2.2.2, the notions of a "liberal candidate" and a "conservative candidate" are well-defined: all pure strategy equilibria have the same candidate take a more liberal position than its opponent. The "liberal candidate" is the one who believes the median voter is more liberal than the median voter according to the beliefs of the "conservative candidate." However, the "liberal candidate" need not make a policy announcement corresponding to the ideal policy of who it believes to be the (relatively liberal) median voter and the "conservative candidate" need not make a policy announcement corresponding to the ideal policy of who it believes to be the (relatively conservative) median voter.

Corollary 2.3 shows that even if there is minimal disagreement between the candidates about voter preferences, in equilibrium the candidate policy announcements can be very far apart.[3]

Corollary 2.3. *Suppose candidates are office-motivated. Suppose* $F_A^{-1}(\frac{1}{2}) < \frac{1}{2}$ *and* $F_B^{-1}(\frac{1}{2}) > \frac{1}{2}$*. Then it is a pure strategy equilibrium for candidate* A *to announce* $\rho_A = \epsilon$ *and for candidate* B *to announce* $\rho_B = 1 - \epsilon$*, for any* $\epsilon \in [0, \frac{1}{2})$*.*

In particular, Corollary 2.3 shows that if the candidates disagree about whether the median voter has an ideal policy above or below the middle of the policy space (i.e., $\frac{1}{2}$),[4] then one candidate announcing policy 0 and the other announcing policy 1 is an equilibrium. Thus,

[3]Note that since F_A and F_B are assumed to have everywhere strictly positive densities, $F_i^{-1}(\frac{1}{2})$ cannot be the endpoints of the policy space, 0 or 1.

[4]Note this "instance" of $\frac{1}{2}$ in the paper concerns the middle of the policy space, which is different from the other "instance" of $\frac{1}{2}$ in the paper which concerns the median voter.

even a very small disagreement can result in big policy differences. Also, note that candidates are not necessarily announcing policies that they believe are ideal for who they believe to be the median voter.

Corollary 2.4. *Suppose candidates are office-motivated. Suppose* $F_A^{-1}(\frac{1}{2}) < F_B^{-1}(\frac{1}{2}) < \frac{1}{2}$*. Then it is a pure strategy equilibrium for candidate* A *to announce* $\rho_A = 0$ *and for candidate* B *to announce* $\rho_B = 2F_B^{-1}(\frac{1}{2}) - \epsilon$*, for sufficiently small* $\epsilon > 0$*.*

Corollary 2.4 shows that there can be big policy differences with very small disagreements about voter preferences, even if candidates agree about whether the median voter has an ideal policy above or below the middle of the policy space (i.e., $\frac{1}{2}$). In the case of Corollary 2.4, both candidates believe that the median voter has an ideal policy more liberal than the middle of the policy space. A similar sort of equilibrium is possible when both candidates believe that the median voter has an ideal policy more conservative than the middle of the policy space. So, for example, if $F_A^{-1}(\frac{1}{2}) = 0.489$ and $F_B^{-1}(\frac{1}{2}) = 0.49$, then it is an equilibrium to have $(\rho_A, \rho_B) = (0, 0.98 - \epsilon)$ for sufficiently small $\epsilon > 0$.

3. Equilibrium with policy-motivated candidates

Instead of wanting to win office, candidates might have preferences over the policy enacted. In this section, candidates are policy-motivated. Each candidate has preferences over enacted policies in the same way as the voters in the previous section. Specifically, the ideal policy of candidates A and B are θ_A and θ_B, respectively. As standard in this literature, candidates credibly commit to their policy announcement, so that the policy announcement is actually enacted upon winning the election.[5]

Each candidate $i \in \{A, B\}$ maximizes expected utility, and has a utility function over the enacted policy ρ, $u_i(\rho)$. The candidates' utility functions are assumed to satisfy two standard conditions: (i) they represent strictly single-peaked preferences, so that if ρ' and ρ''

[5] A different literature on "citizen candidates" following Osborne and Slivinski (1996), Besley and Coate (1997), and Besley and Coate (1998), allows winning candidates to implement a different policy. This literature has been developed further in, for example, Cadigan and Janeba (2002), Dhillon and Lockwood (2002), Cadigan (2005), Usher (2005), Brusco and Roy (2011), and Aytimur, Boukouras, and Schwager (2016).

are policies such that $|\rho' - \theta_i| < |\rho'' - \theta_i|$ then $u_i(\rho') > u_i(\rho'')$. And, (ii) they are continuous functions.

Lemma 3.1 establishes necessary conditions for policy announcements to be a pure strategy equilibrium. This provides some insight into the decision problem facing each candidate. Based on that, Theorem 3.2 fully characterizes the set of pure strategy equilibria.

Lemma 3.1. *Suppose candidates are policy-motivated. If (ρ_A, ρ_B) is a pure strategy equilibrium of policy announcements, then:*

(1) If either candidate believes it will win, then it must announce its ideal policy.

In other words, if a candidate $i \in \{A, B\}$ has beliefs such that $F_i(\frac{\rho_A+\rho_B}{2}) > \frac{1}{2}$ and $\rho_i < \rho_j$ or $F_i(\frac{\rho_A+\rho_B}{2}) < \frac{1}{2}$ and $\rho_i > \rho_j$, then $\rho_i = \theta_i$.

(2) If a candidate $i \in \{A, B\}$ believes it will lose, then candidate i must have an ideal policy and beliefs that imply either: that it believes that it cannot change the enacted policy (because the other candidate has already announced the ideal policy of the median voter according to the beliefs of candidate i), or that it believes that it could win or tie the election with a different policy announcement, but only for other policy announcements that move the enacted policy away from its ideal policy.

In other words, if a candidate i has beliefs such that $F_i(\frac{\rho_A+\rho_B}{2}) < \frac{1}{2}$ and $\rho_i < \rho_j$ or $F_i(\frac{\rho_A+\rho_B}{2}) > \frac{1}{2}$ and $\rho_i > \rho_j$, then either $\rho_j = F_i^{-1}(\frac{1}{2})$; or $\rho_j > F_i^{-1}(\frac{1}{2})$ and $\theta_i \geq \rho_j$; or $\rho_j < F_i^{-1}(\frac{1}{2})$ and $\theta_i \leq \rho_j$.

(3) If the candidates announce different policies, then neither candidate can believe the election will be a tie.

In other words, for policy announcements $\rho_A \neq \rho_B$, it must be that $F_A(\frac{\rho_A+\rho_B}{2}) \neq \frac{1}{2}$ and $F_B(\frac{\rho_A+\rho_B}{2}) \neq \frac{1}{2}$.

(4) If the candidates announce the same policy ρ, it cannot be that either candidate believes the median voter's ideal policy and that candidate's own ideal policy are on the same side of ρ.

In other words, for policy announcements $\rho_A = \rho_B = \rho$, for each candidate $i \in \{A, B\}$, it cannot be that $F_i^{-1}(\frac{1}{2}) < \rho$ and $\theta_i < \rho$ and it cannot be that $F_i^{-1}(\frac{1}{2}) > \rho$ and $\theta_i > \rho$.

Theorem 3.2. *Suppose candidates are policy-motivated. A pure strategy equilibrium exists. Policy announcements (ρ_A, ρ_B) are a pure strategy equilibrium if and only if for each candidate $i \in \{A, B\}$, one of the following holds:*

(3.2.1) $F_i^{-1}(\frac{1}{2}) < \frac{\rho_A+\rho_B}{2}$ and $\rho_i < \rho_j$ and $\rho_i = \theta_i$. (i believes it will win, with its ideal policy)

(3.2.2) $F_i^{-1}(\frac{1}{2}) > \frac{\rho_A+\rho_B}{2}$ and $\rho_i > \rho_j$ and $\rho_i = \theta_i$. (i believes it will win, with its ideal policy)

(3.2.3) $F_i^{-1}(\frac{1}{2}) > \frac{\rho_A+\rho_B}{2}$ and $\rho_i < \rho_j$ and $\rho_j = F_i^{-1}(\frac{1}{2})$. (i believes it will lose, but cannot change policy)

(3.2.4) $F_i^{-1}(\frac{1}{2}) > \frac{\rho_A+\rho_B}{2}$ and $\rho_i < \rho_j$ and $\rho_j > F_i^{-1}(\frac{1}{2})$ and $\theta_i \geq \rho_j$. (i believes it will lose, but cannot improve policy)

(3.2.5) $F_i^{-1}(\frac{1}{2}) > \frac{\rho_A+\rho_B}{2}$ and $\rho_i < \rho_j$ and $\rho_j < F_i^{-1}(\frac{1}{2})$ and $\theta_i \leq \rho_j$. (i believes it will lose, but cannot improve policy)

(3.2.6) $F_i^{-1}(\frac{1}{2}) < \frac{\rho_A+\rho_B}{2}$ and $\rho_i > \rho_j$ and $\rho_j = F_i^{-1}(\frac{1}{2})$. (i believes it will lose, but cannot change policy)

(3.2.7) $F_i^{-1}(\frac{1}{2}) < \frac{\rho_A+\rho_B}{2}$ and $\rho_i > \rho_j$ and $\rho_j > F_i^{-1}(\frac{1}{2})$ and $\theta_i \geq \rho_j$. (i believes it will lose, but cannot improve policy)

(3.2.8) $F_i^{-1}(\frac{1}{2}) < \frac{\rho_A+\rho_B}{2}$ and $\rho_i > \rho_j$ and $\rho_j < F_i^{-1}(\frac{1}{2})$ and $\theta_i \leq \rho_j$. (i believes it will lose, but cannot improve policy)

(3.2.9) $\rho_A = \rho_B$ and it is neither the case that $F_i^{-1}(\frac{1}{2}) < \rho_A = \rho_B$ and $\theta_i < \rho_A = \rho_B$ nor the case that $F_i^{-1}(\frac{1}{2}) > \rho_A = \rho_B$ and $\theta_i > \rho_A = \rho_B$. (candidates announce the same policy, and cannot improve policy)

Therefore, the actual set of equilibria depends on the primitives of the model (the specifications of F_i and θ_i). Some pairs of the conditions in Theorem 3.2 are necessarily incompatible across candidates A and B, for any specification of the model primitives. For example, Condition 3.2.1 cannot hold for both candidates, since that would require the inconsistent inequalities $\rho_A < \rho_B$ and $\rho_B < \rho_A$. However, for example, it is possible that Condition 3.2.1 holds for candidate $i = A$ and Condition 3.2.2 holds for candidate $i = B$; in turn, this requires the condition on the primitives that $F_A^{-1}(\frac{1}{2}) < F_B^{-1}(\frac{1}{2})$.

Corollary 3.3 below provides an example of how different primitives can lead to different equilibria. Corollary 3.3 shows that, depending on the arrangements of model primitives, there exist equilibria in which both candidates believe they will win the election and announce their ideal policy, equilibria in which one candidate believes it will win and announces its ideal policy and the other believes it will lose, and equilibria in which both candidates believe they will lose. Overall, the wide range of equilibria in Corollary 3.3 shows that equilibrium with policy-motivated candidates can be quite sensitive to disagreement about the voters. With even a small amount of disagreement about the median voter, policy-motivated candidates can announce quite different policies in equilibrium.

Corollary 3.3. *Suppose candidates are policy-motivated. Let $\theta_A < \theta_B$. The following pure strategy equilibria exist under the specified conditions on F_A and F_B.*

(3.3.1) There is an equilibrium in which both candidates believe they win the election, if $\rho_A = \theta_A < F_A^{-1}(\frac{1}{2}) < \frac{\rho_A+\rho_B}{2} = \frac{\theta_A+\theta_B}{2} < F_B^{-1}(\frac{1}{2}) < \rho_B = \theta_B$.

(3.3.2) There is an equilibrium in which one candidate believes it wins, and the other candidate believes it loses, if $\rho_A = \theta_A = F_B^{-1}(\frac{1}{2}) < F_A^{-1}(\frac{1}{2}) < \frac{\rho_A+\rho_B}{2} < \rho_B$. In this equilibrium, candidate A believes it wins and the policy θ_A is enacted, while candidate B believes it loses and the policy θ_A is enacted.

(3.3.3) There is an equilibrium in which both candidates believe they lose, if $\rho_A = F_B^{-1}(\frac{1}{2}) < \rho_B = F_A^{-1}(\frac{1}{2})$.

Equilibrium 3.3.1 concerns an empirically plausible arrangement of model primitives where policy-motivated candidates have more extreme policy preferences than voters, with each candidate's beliefs about the voters in line (relative to the other candidate's beliefs) with its own policy preference. That is, candidate A has a liberal ideal policy and believes the median voter is relatively liberal; candidate B has a conservative ideal policy and believes the median voter is relatively conservative.

In Equilibrium 3.3.1, both candidates announce their ideal policy, regardless of the magnitude of the difference between these ideal policies. For a numerical example, suppose that $F_A^{-1}(\frac{1}{2}) = 0.49$ and $F_B^{-1}(\frac{1}{2}) = 0.51$, so candidate A believes that the median voter is slightly more liberal than believed by candidate B. Then it would be an equilibrium for both candidates to announce their ideal policy, as long as the average of their ideal policies is around 0.5 (i.e., between 0.49 and 0.51). The full set of equilibria for this arrangement of model primitives is given in Corollary 3.4. In other models with policy-motivated candidates, except with shared beliefs about the voters, similar arrangements of model primitives with candidate ideal points on opposite sides of the "median voter" has been paid attention to, for example in Duggan and Fey (2005).

However, this is not the unique equilibrium for this arrangement of model primitives, as Corollary 3.4 below demonstrates. Specifically, there are also equilibria in which both candidates believe they will lose, believing that the other candidate will enact a policy not too far away from the losing candidate's ideal policy.

Corollary 3.4. *Suppose candidates are policy-motivated. Suppose that $\theta_A < F_A^{-1}(\frac{1}{2}) < F_B^{-1}(\frac{1}{2}) < \theta_B$. Then a pure strategy equilibrium exists, and the only pure strategy equilibria are:*

(3.4.1) $\rho_A = \theta_A$ and $\rho_B = \theta_B$ when $\rho_A = \theta_A < F_A^{-1}(\frac{1}{2}) < \frac{\rho_A+\rho_B}{2} = \frac{\theta_A+\theta_B}{2} < F_B^{-1}(\frac{1}{2}) < \rho_B = \theta_B$

(3.4.2) any (ρ_A, ρ_B) that satisfy $\theta_A \leq \rho_B \leq F_A^{-1}(\frac{1}{2}) < \frac{\rho_A+\rho_B}{2} < F_B^{-1}(\frac{1}{2}) \leq \rho_A \leq \theta_B$

Corollaries 3.3 and 3.4 cover the case that $\theta_A < F_A^{-1}(\frac{1}{2}) < F_B^{-1}(\frac{1}{2}) < \theta_B$. As noted above, this concerns the empirically plausible scenario where candidates believe that the median voter is relatively "in line" with their own ideal policies. Because Theorem 3.2 fully characterizes the set of equilibria for any specification of model primitives, it is possible to consider other specifications.

Another arrangement is $\theta_A < F_B^{-1}(\frac{1}{2}) < F_A^{-1}(\frac{1}{2}) < \theta_B$. This case corresponds to a scenario where candidates believe that the median voter is relatively "out of line" with their own ideal policies. This also seems empirically plausible. For example, there is evidence that people perceive their political adversaries as "more extreme" than they really are (e.g., Levendusky and Malhotra (2016) and Ahler and Sood (2018)). Plausibly, this could lead the "liberal" candidate to believe that the median voter is "more conservative" than it actually is, and it could lead the "conservative" candidate to believe that the median voter is "more liberal" than it actually is. In this case, candidates could have beliefs that are actually "out of line" with their own ideal policies. Corollary 3.5 below characterizes the equilibria in this case.

Corollary 3.5. *Suppose candidates are policy-motivated. Suppose that* $\theta_A < F_B^{-1}(\frac{1}{2}) < F_A^{-1}(\frac{1}{2}) < \theta_B$. *Then a pure strategy equilibrium exists, and the only pure strategy equilibria are:*

(3.5.1) $\rho_A = \rho_B = \rho$ *that satisfy* $F_B^{-1}(\frac{1}{2}) \leq \rho \leq F_A^{-1}(\frac{1}{2})$

(3.5.2) any (ρ_A, ρ_B) *that satisfy* $F_B^{-1}(\frac{1}{2}) < \frac{\rho_A+\rho_B}{2} < F_A^{-1}(\frac{1}{2})$ *and* $F_B^{-1}(\frac{1}{2}) \leq \rho_A \leq \theta_B$ *and* $\theta_A \leq \rho_B \leq F_A^{-1}(\frac{1}{2})$ *and* $\rho_A < \rho_B$

Equilibrium 3.5.1 actually does roughly approximate the prediction of the "median voter theorem," in the sense that the candidates make the *same* policy announcement, and that policy announcement is "in between" the ideal policies believed by the candidates to be held by the median voter. Therefore, in this situation where candidates believe the voters are relatively "out of line" with their own ideal policies, the candidates *can* approximate the behavior predicted by the "median voter theorem" in equilibrium. However, this is *not* the only equilibrium.

Equilibrium 3.5.2 resembles Equilibrium 3.4.2. In this equilibrium, both candidates believe they will lose, believing that the other candidate will win and enact a policy not too far away from the losing candidate's ideal policy. Essentially, both candidates think they are "tricking" the other candidate into winning the election after announcing a policy that will be close to their own ideal policy. This equilibrium allows for *large* policy gaps even for *small* amounts of disagreement between the candidates about the voters. For example, consider $F_B^{-1}(\frac{1}{2}) = 0.49$, and $F_A^{-1}(\frac{1}{2}) = 0.51$. Then it would be an equilibrium for both candidates to announce the ideal policies of the *other* candidate, $\rho_A = \theta_B$ and $\rho_B = \theta_A$, as long as the average of the ideal policies is around 0.50 (i.e., between 0.49 and 0.51).

Alternatively, if the candidates share beliefs about the preferences of voters, with one candidate having ideal policy more liberal than the median voter and the other candidate having ideal policy more conservative than the median voter, then the "median voter theorem" prediction holds, per Corollary 3.6 below. This result is already known in the literature in this case of shared beliefs with policy-motivated candidates (e.g., Wittman (1977), Calvert (1985), and Duggan and Fey (2005)). Corollary 3.6 is presented here simply for point of reference for the other results in this paper.

Corollary 3.6 (Standard median voter theorem with policy-motivated candidates)**.** *Suppose candidates are policy-motivated. Suppose that $F_A = F = F_B$ and $\theta_A < F^{-1}(\frac{1}{2}) < \theta_B$. Then the only pure strategy equilibrium is $\rho_A = F^{-1}(\frac{1}{2}) = \rho_B$.*

Therefore, considering the case considered in Corollary 3.4, with a small amount of disagreement about the preferences of the median voter, the equilibrium switches from the classical prediction of the median voter theorem that both candidates locate at the ideal policy of the median voter (Corollary 3.6) to either both candidates announcing their ideal policy (Equilibrium 3.4.1), or announcing a policy close to the other candidate's ideal policy (Equilibrium 3.4.2). The case considered in Corollary 3.5 allows for the *possibility* that the candidates approximate the predictions of the "median voter theorem;" but, this is not the only equilibrium in this case.

Overall, this shows the *high sensitivity* of the predictions to disagreement about the preferences of the voters, with policy-motivated candidates. Corollaries 2.3 and 2.4 show similar results for office-motivated candidates. Collectively, this shows that the predictions of the median voter theorem are very sensitive to disagreement about the preferences of voters.

4. Discussion

We can compare the results in this paper to other results in the literature that explore related issues about information. One existing approach allows that candidates have private information about the preferences of the voters, while maintaining the standard assumption that candidates have a common prior about the preferences of voters. In contrast, this paper relaxes the common prior assumption to allow candidates to have completely different beliefs about the preferences of voters. Bernhardt, Duggan, and Squintani (2007) study a model in which office-motivated candidates receive private signals about the preferences of the voters, maintaining the standard assumption of a common prior. The "median voter theorem" essentially still happens in the setup of Bernhardt, Duggan, and Squintani (2007). Essentially, when there is just a bit of private information candidates must locate in equilibrium as (almost) pure strategies at the medians according to the distributions induced by their signals, which is not too far from the standard median voter theorem in case the induced distributions are not too different.[6] Similar results have been found in similar setups, including in Calvert (1985) and Duggan (2006). Kikuchi (2016) studies a model of vote-motivated candidates who have different beliefs about the voters (but probabilistic voting rather than Downsian voting), and essentially finds again the "median voter theorem" basically holds.[7]

Unlike in those setups, the results in the setup of this paper show considerably more equilibria, some of which are very far from the prediction of the median voter theorem even if the difference between the beliefs of the candidates is small. All of those papers, and this paper, share the part of the modeling approach that allows beliefs that are not necessarily

[6]Bernhardt, Duggan, and Squintani (2007) find that in any pure strategy equilibrium of this game, after observing any given signal, a candidate announces the ideal policy of the median of the distribution describing the location of the median voter, where this distribution is the one given by the observed signal and the opponent observing the same signal. Therefore, in particular, the candidates take the same policy position conditional on observing the same signal, although not necessarily that of the "true" median voter. Unlike the results in this paper, Bernhardt, Duggan, and Squintani (2007) show by example that a pure strategy equilibrium does not always exist. However, Bernhardt, Duggan, and Squintani (2007) show that any mixed strategy equilibrium must be close to the pure strategy equilibrium, placing most mass at the same conditional median as a pure strategy equilibrium, as long as there is not too much private information.

[7]Specifically, Kikuchi (2016) finds that a Nash equilibrium may not exist, but if it does, it involves the candidates announcing the ideal policy of a suitably defined "median voter." In the case of an approximate equilibrium, the candidates announce similar policies.

consistent with "realized" voting behavior. Imposing the requirement that beliefs are also consistent with "realized" voting behavior would essentially shut down any possibility of candidates having different beliefs.

Other papers have explored other sources of policy divergence in equilibrium. One possibility is candidate valence. For example, Ansolabehere and Snyder (2000), Groseclose (2001), Aragonès and Palfrey (2002), Hummel (2010), and Aragonès and Xefteris (2012) have results that show that candidate valence can induce different policy announcements; but, as discussed in those papers, small valence induces small gaps in the policy announcements, basically recovering the median voter theorem. Another possibility is that voters use the policy announcements of the candidates to learn about their own ideal policies. This concerns the information of the voters rather than the candidates. For example, Martinelli (2001) has results that show that voter-learning can induce different policy announcements (when candidates have policy-motivations); but, if voters are highly informed apart from learning from candidate policy announcements, basically the median voter theorem can be recovered.

In the above papers, the "median voter theorem" is generally found to be relatively "robust" to small deviations from the standard spatial voting model setup. By contrast, this paper shows that even a small amount of disagreement among the candidates about the voters can result in a large policy gap. This suggests that candidates having different beliefs about the voters could plausibly explain large policy gaps observed in real-world elections.

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Appendix A. Proofs

This appendix provides the proofs.

Proof of Lemma 2.1. $F_A^{-1}(\frac{1}{2}) \neq F_B^{-1}(\frac{1}{2})$ is equivalent to $F_A^{-1}(\frac{1}{2}) > F_B^{-1}(\frac{1}{2})$ or $F_A^{-1}(\frac{1}{2}) < F_B^{-1}(\frac{1}{2})$.

Consider the former inequality. This is equivalent to existence of ρ such that $F_B^{-1}(\frac{1}{2}) < \rho < F_A^{-1}(\frac{1}{2})$, which is equivalent to $\frac{1}{2} < F_B(\rho)$ and $F_A(\rho) < \frac{1}{2}$.

Consider the latter inequality. This is equivalent to existence of ρ such that $F_A^{-1}(\frac{1}{2}) < \rho < F_B^{-1}(\frac{1}{2})$, which is equivalent to $\frac{1}{2} < F_A(\rho)$ and $F_B(\rho) < \frac{1}{2}$. □

Proof of Theorem 2.2. Consider the case in Equilibrium 2.2.1. That is, suppose that $\rho_A < \rho_B$, and suppose that $F_A(\frac{\rho_A+\rho_B}{2}) > \frac{1}{2}$ and $F_B(\frac{\rho_A+\rho_B}{2}) < \frac{1}{2}$. The case in Equilibrium 2.2.2 follows similarly.

Since $\rho_A < \rho_B$, candidate A believes that all voters more liberal than $\frac{\rho_A+\rho_B}{2}$ will vote for A; and candidate B believes that all voters more conservative than $\frac{\rho_A+\rho_B}{2}$ will vote for B. Because beliefs have a density by assumption, the case of indifferent voters is irrelevant. Therefore, candidate A believes it will get share $F_A(\frac{\rho_A+\rho_B}{2})$ of the vote, and candidate B believes it will get share $1 - F_B(\frac{\rho_A+\rho_B}{2})$ of the vote. Consequently, both candidates believe that they will win the election, so there is no incentive to deviate. Therefore, this is a pure strategy equilibrium.

Furthermore, for policy announcements $\rho_A < \rho_B$ to be an equilibrium, it must be that $F_A(\frac{\rho_A+\rho_B}{2}) > \frac{1}{2}$ and $F_B(\frac{\rho_A+\rho_B}{2}) < \frac{1}{2}$. First, no pure strategy equilibrium can have either that $F_A(\frac{\rho_A+\rho_B}{2}) < \frac{1}{2}$ or $F_B(\frac{\rho_A+\rho_B}{2}) > \frac{1}{2}$. In either such case, at least one candidate believes it will lose the election for sure and therefore that candidate has a profitable deviation to announcing the same policy as its opponent, giving it a $\frac{1}{2}$ probability of winning the election. Also, no pure strategy equilibrium can have either $F_A(\frac{\rho_A+\rho_B}{2}) = \frac{1}{2}$ or $F_B(\frac{\rho_A+\rho_B}{2}) = \frac{1}{2}$. In the former such case, by strictly positive density, $F_A(\rho_B) > \frac{1}{2}$ since $\rho_B > \frac{\rho_A+\rho_B}{2}$, so candidate A has an incentive to announce a policy just on the liberal side of ρ_B. In the latter such case, $F_B(\rho_A) < \frac{1}{2}$ since $\rho_A < \frac{\rho_A+\rho_B}{2}$, so candidate B has an incentive to announce a policy just on the conservative side of ρ_A.

Consequently, pure strategy equilibrium with $\rho_A < \rho_B$ must have $F_A(\frac{\rho_A+\rho_B}{2}) > \frac{1}{2}$ and $F_B(\frac{\rho_A+\rho_B}{2}) < \frac{1}{2}$, and all $\rho_A < \rho_B$ with this property are pure strategy equilibria.

Consider the case in Equilibrium 2.2.3. That is, suppose that $\rho_A = \rho = \rho_B$. If $F_A(\rho) = \frac{1}{2} = F_B(\rho)$, then neither candidate has an incentive to deviate, so this is an equilibrium. If either candidate announced a different policy, they would believe that they would lose the election, rather than tie. However, if $F_i(\rho) \neq \frac{1}{2}$ for some candidate i then candidate i has an incentive to deviate slightly. If $F_i(\rho) < \frac{1}{2}$, then candidate i has an incentive to deviate to a slightly more conservative policy. If $F_i(\rho) > \frac{1}{2}$, then candidate i has an incentive to deviate to a slightly more liberal policy.

To see that there is always a pure strategy equilibrium, note that if there is ρ such that $F_A(\rho) = \frac{1}{2} = F_B(\rho)$, then $\rho_A = \rho = \rho_B$ is an equilibrium. Otherwise, by Lemma 2.1, there is ρ such that $F_i(\rho) < \frac{1}{2}$ and $F_j(\rho) > \frac{1}{2}$, for some arrangement of the indexing. Then let $\rho_i = \rho + \epsilon$ and $\rho_j = \rho - \epsilon$, so that $\rho_i > \rho_j$. It follows that $\frac{\rho_A+\rho_B}{2} = \rho$, so $F_i(\frac{\rho_A+\rho_B}{2}) < \frac{1}{2}$ and $F_j(\frac{\rho_A+\rho_B}{2}) > \frac{1}{2}$, so this is an equilibrium.

Finally, to see that all pure strategy equilibria have the same ordering of policy announcements: if there is ρ such that $F_A(\rho) = \frac{1}{2} = F_B(\rho)$, then there cannot be ρ' such that $F_A(\rho') > (<)\frac{1}{2}$ and $F_B(\rho') < (>)\frac{1}{2}$, since F_A and F_B are strictly increasing. Thus, if there is an equilibrium of the form in Equilibrium 2.2.3, there cannot be an equilibrium in the form of Equilibria 2.2.1 and 2.2.2. Similarly, since F_A and F_B are strictly increasing, if there is ρ' such that $F_A(\rho') > \frac{1}{2}$ and $F_B(\rho') < \frac{1}{2}$, then there can not be ρ'' such that the sense of the inequalities are (weakly) switched. Thus, if there is an equilibrium of the form in Equilibrium 2.2.2, there cannot be an equilibrium in the form of Equilibria 2.2.1 and 2.2.3. And similarly, if there is an equilibrium of the form in Equilibrium 2.2.1, there cannot be an equilibrium in the form of Equilibria 2.2.2 and 2.2.3.

Note that Equilibria 2.2.1 and 2.2.2 happen only when candidates disagree about the median voter, given the characterization in Lemma 2.1. And Equilibrium 2.2.3 happens only when candidates agree about the median voter. □

Proof of Corollary 2.3. Note that $\rho_A < \rho_B$ and $\frac{\rho_A+\rho_B}{2} = \frac{1}{2}$. And note that $F_A(\frac{1}{2}) > \frac{1}{2}$ and $F_B(\frac{1}{2}) < \frac{1}{2}$. This is Equilibrium 2.2.1. □

Proof of Corollary 2.4. Let $a = F_A^{-1}(\frac{1}{2})$ and $b = F_B^{-1}(\frac{1}{2})$. At exactly $(\rho_A, \rho_B) = (0, 2b)$, it holds that $F_A(\frac{\rho_A+\rho_B}{2}) = F_A(b) > F_A(a) = \frac{1}{2}$ and $F_B(\frac{\rho_A+\rho_B}{2}) = F_B(b) = \frac{1}{2}$. Therefore, using continuity of the beliefs, at $(\rho_A, \rho_B) = (0, 2b - \epsilon)$ for sufficiently small $\epsilon > 0$, the sense of the inequality concerning F_A is unchanged, and the equality concerning F_B becomes a strict inequality with $F_B(\frac{\rho_A+\rho_B}{2}) < \frac{1}{2}$. This is Equilibrium 2.2.1. □

Proof of Lemma 3.1. Proof of Lemma 3.1.1: Candidate i can move its policy announcement marginally in the direction of its ideal policy, believing that it will still win since F_i is assumed to have a density, and be strictly better off with the new policy announcement given strictly single-peaked preferences.

Proof of Lemma 3.1.2: Let the candidate who believes it will lose be candidate i, with candidate j being the other candidate. There are three cases to consider, depending on the ordering of ρ_j and $F_i^{-1}(\frac{1}{2})$.

If $\rho_j = F_i^{-1}(\frac{1}{2})$, or equivalently $F_i(\rho_j) = \frac{1}{2}$, then candidate i believes that it cannot change the enacted policy given that candidate j announces the policy ρ_j. If candidate i deviates to also announce ρ_j, then the election will be tied, but ρ_j will be enacted since both candidates announced it. Alternatively, if candidate i deviates to any other policy, candidate i believes it will continue to lose the election, in which case ρ_j will be enacted. Therefore, candidate i does not have a profitable deviation.

If $\rho_j > F_i^{-1}(\frac{1}{2})$, or equivalently $F_i(\rho_j) > \frac{1}{2}$, then candidate i believes that it will win the election with a policy $\rho_j - \epsilon$ for small enough $\epsilon > 0$, slightly more liberal than ρ_j. Also, candidate i believes that it will (continue to) lose the election with any policy announcement that is more conservative than ρ_j. And, candidate i believes that it would tie the election with the policy announcement ρ_j. In the latter two cases, ρ_j will be enacted regardless, so this cannot be a profitable deviation for candidate i. In the former case, candidate i gets utility $u_i(\rho_j)$ under the status quo. In the case of deviating to the slightly more liberal policy announcement, candidate i would get utility $u_i(\rho_j - \epsilon)$. In the case that $\theta_i < \rho_j$, then taking ϵ small enough so that $\theta_i < \rho_j - \epsilon < \rho_j$, by strictly single-peaked preferences, it would follow that $u_i(\rho_j - \epsilon) > u_i(\rho_j)$, so this would be a profitable deviation. Therefore, for this to not be a profitable deviation, it must be that $\theta_i \geq \rho_j$. Therefore, if $\theta_i < \rho_j$, candidate i has a profitable deviation, and does not otherwise.

Similarly, if $\rho_j < F_i^{-1}(\frac{1}{2})$, or equivalently $F_i(\rho_j) < \frac{1}{2}$, then candidate i has a profitable deviation if $\theta_i > \rho_j$, and does not otherwise.

Proof of Lemma 3.1.3: Suppose candidate i believes the election will result in a tie, and the candidates do not make the same policy announcement. Consider the case that $\rho_i < F_i^{-1}(\frac{1}{2})$. The case that $\rho_i > F_i^{-1}(\frac{1}{2})$ is similar. The case that $\rho_i = F_i^{-1}(\frac{1}{2})$ is ruled out, because candidate i would believe it would win the election in this case unless $\rho_i = \rho_j$, which is directly ruled out.

Since candidate i believes the election will result in a tie, it must be that $F_i^{-1}(\frac{1}{2}) < \rho_j$. This is because $\rho_j < \rho_i$ would imply that candidate i believes that candidate i will win the election, and $\rho_i < \rho_j \leq F_i^{-1}(\frac{1}{2})$ would imply that candidate i believes that candidate j will win the election. And $\rho_i \neq \rho_j$ is directly ruled out.

It cannot be that θ_i is such that the ordering $\theta_i \leq \rho_i < F_i^{-1}(\frac{1}{2}) < \rho_j$ holds, because in that case, candidate i has a profitable deviation to announce a slightly more conservative policy, and believe it will win for sure. Prior to this deviation, candidate i's expected utility is $\frac{1}{2}u_i(\rho_i) + \frac{1}{2}u_i(\rho_j)$. After this deviation, candidate i's expected utility is $u_i(\rho_i + \epsilon)$. By strictly single-peaked preferences, and the fact that ρ_i is closer to θ_i, as compared to ρ_j, $u_i(\rho_i) > u_i(\rho_j)$. By continuity of the utility functions, for small enough $\epsilon > 0$, also $|u_i(\rho_i + \epsilon) - u_i(\rho_i)| < \frac{1}{2}(u_i(\rho_i) - u_i(\rho_j))$. Also, by strictly singled-peaked preferences, and the fact that ρ_i is to closer to θ_i, as compared to $\rho_i + \epsilon$, $u_i(\rho_i) > u_i(\rho_i + \epsilon)$. Therefore,

$u_i(\rho_i+\epsilon)-u_i(\rho_i) > -\frac{1}{2}(u_i(\rho_i)-u_i(\rho_j))$. Equivalently, $u_i(\rho_i+\epsilon) > u_i(\rho_i)+\frac{1}{2}(u_i(\rho_j)-u_i(\rho_i)) = \frac{1}{2}u_i(\rho_i) + \frac{1}{2}u_i(\rho_j)$. Consequently, this would be a profitable deviation for candidate i.

Also, it cannot be that $\theta_i \in (\rho_i, \rho_j]$, because then candidate i can announce and believe it will win with its ideal policy, a profitable deviation. After this deviation, candidate i's expected utility is $u_i(\theta_i)$, which obviously exceeds its expected utility prior to this deviation, given strictly single-peaked preferences. And it cannot be that $\theta_i > \rho_j$, because then it is a profitable deviation for candidate i to also announce ρ_j, resulting in it being enacted for sure, thereby avoiding the possibility of less-preferred ρ_i being enacted in the tie. After this deviation, candidate i's expected utility is $u_i(\rho_j)$. In this arrangement, $|\rho_j - \theta_i| < |\rho_i - \theta_i|$, so by strictly single-peaked preferences, $u_i(\rho_j) > u_i(\rho_i)$, so this is indeed a profitable deviation. Therefore, in equilibrium, candidate i cannot believe the election will result in a tie, when the candidates do not make the same policy announcement.

Proof of Lemma 3.1.4: In the former case, candidate i believes it would win the election with a policy announcement slightly more liberal than ρ. By strictly single-peaked preferences, this would increase candidate i's utility, in the case that $\theta_i < \rho$. Thus, the former case is inconsistent with equilibrium. By similar arguments, the latter case is also inconsistent with equilibrium. □

Proof of Theorem 3.2. By the following logic, the claims in Lemma 3.1 establish the necessity of the characterizations of the policy announcements, based on a case analysis.

For a given candidate $i \in \{A, B\}$, any arrangement of policy announcements is such that exactly one of the following holds: (i) $F_i^{-1}(\frac{1}{2}) < \frac{\rho_A+\rho_B}{2}$ or (ii) $F_i^{-1}(\frac{1}{2}) > \frac{\rho_A+\rho_B}{2}$ or (iii) $F_i^{-1}(\frac{1}{2}) = \frac{\rho_A+\rho_B}{2}$. And exactly one of the following holds (a) $\rho_i < \rho_j$ or (b) $\rho_i > \rho_j$ or (c) $\rho_i = \rho_j$.

For any case where candidate i believes it will win the election (i.e., cases i-a and ii-b), it must announce its ideal policy, by Lemma 3.1.1. This establishes that, in these cases, the conditions in Conditions 3.2.1 and 3.2.2 are necessary.

For any case where candidate i believes it will lose the election (i.e., cases i-b and ii-a), it must be unable to announce a different policy closer to its ideal policy and win or tie, by Lemma 3.1.2. Exactly one of the following holds: (A) $\rho_j = F_i^{-1}(\frac{1}{2})$ or (B) $\rho_j > F_i^{-1}(\frac{1}{2})$ or (C) $\rho_j < F_i^{-1}(\frac{1}{2})$. In case (A), candidate i believes that the policy ρ_j will be enacted regardless of what candidate i does; therefore, there are no necessary conditions on θ_i implied by equilibrium. This gives the conditions in Conditions 3.2.3 and 3.2.6. In case (B), candidate i believes that it could win with a policy announcement just on the liberal side of ρ_j. Prior to any deviation, candidate i believes it will lose the election and get utility $u_i(\rho_j)$. After that deviation, candidate i gets utility $u_i(\rho_j - \epsilon)$ for small $\epsilon > 0$. If $\theta_i < \rho_j$, then $\rho_j - \epsilon - \theta_i = |\rho_j - \epsilon - \theta_i| < |\rho_j - \theta_i| = \rho_j - \theta_i$ for small enough $\epsilon > 0$. And hence by strictly single-peaked preference, $u_i(\rho_j - \epsilon) > u_i(\rho_j)$, making this a profitable deviation. Thus, it must be that $\theta_i \geq \rho_j$ as a necessary condition for this to be an equilibrium. This gives the conditions in Conditions 3.2.4 and 3.2.7. By similar logic, case (C) gives the conditions in Conditions 3.2.5 and 3.2.8.

The remaining cases are i-c, ii-c, iii-a, iii-b, and iii-c. In any case involving iii, candidate i believes the election will be a tie. Therefore, by Lemma 3.1.3, in equilibrium it must be that $\rho_A = \rho_B$. Therefore, iii-a and iii-b are inconsistent with equilibrium. Consequently, only cases i-c, ii-c, and iii-c are potentially consistent with equilibrium. In all cases, Lemma 3.1.4 applies, which gives the conditions in Condition 3.2.9.

The sufficiency of these conditions for equilibrium can be checked directly; indeed, the previous arguments already did so. A pure strategy equilibrium always exists because the policy announcements $\rho_A = F_B^{-1}(\frac{1}{2})$ and $\rho_B = F_A^{-1}(\frac{1}{2})$ is always a pure strategy equilibrium. In the case that $\rho_A = \rho_B$, this satisfies Condition 3.2.9 for both candidates. In the case that $\rho_A < \rho_B$, this satisfies Condition 3.2.3 for candidate A and Condition 3.2.6 for candidate B. In the case that $\rho_A > \rho_B$, this satisfies Condition 3.2.6 for candidate A and Condition 3.2.3 for candidate B. $\square$

Proof of Corollary 3.3. Equilibrium 3.3.1 follows from Condition 3.2.1 for $i = A$ and Condition 3.2.2 for $i = B$.

Equilibrium 3.3.2 follows from Condition 3.2.1 for $i = A$ and Condition 3.2.6 for $i = B$.

Equilibrium 3.3.3 follows from Condition 3.2.3 for $i = A$ and Condition 3.2.6 for $i = B$. □

Proof of Corollary 3.4. Equilibrium 3.4.1 was established already in Equilibrium 3.3.1.

Equilibrium 3.4.2 encompasses multiple cases depending on whether the inequalities involving ρ are equalities or strict inequalities:

(1) If $\rho_B = F_A^{-1}(\frac{1}{2}) < \frac{\rho_A+\rho_B}{2} < F_B^{-1}(\frac{1}{2}) = \rho_A$, this is an equilibrium by Condition 3.2.6 for $i = A$ and Condition 3.2.3 for $i = B$.

(2) If $\rho_B = F_A^{-1}(\frac{1}{2}) < \frac{\rho_A+\rho_B}{2} < F_B^{-1}(\frac{1}{2}) < \rho_A \leq \theta_B$, this is an equilibrium by Condition 3.2.6 for $i = A$ and Condition 3.2.4 for $i = B$.

(3) If $\theta_A \leq \rho_B < F_A^{-1}(\frac{1}{2}) < \frac{\rho_A+\rho_B}{2} < F_B^{-1}(\frac{1}{2}) = \rho_A$, this is an equilibrium by Condition 3.2.8 for $i = A$ and Condition 3.2.3 for $i = B$.

(4) If $\theta_A \leq \rho_B < F_A^{-1}(\frac{1}{2}) < \frac{\rho_A+\rho_B}{2} < F_B^{-1}(\frac{1}{2}) < \rho_A \leq \theta_B$, this is an equilibrium by Condition 3.2.8 for $i = A$ and Condition 3.2.4 for $i = B$.

The following provides a complete analysis of all possible combinations of the necessary conditions for pure strategy equilibrium from Theorem 3.2, where the first number is the condition for candidate A and the second number is the condition for candidate B. It shows the reason that certain combinations are not possible, thereby establishing that the stated equilibria are indeed the only equilibria. The following uses the condition that $\theta_A < \theta_B$.

(1) (Condition 3.2.1, Conditions 3.2.1, 3.2.3 to 3.2.5 and 3.2.9): Requires contradictory orderings of ρ_A and ρ_B. Therefore, cannot be an equilibrium for any specification of model primitives.

(2) (Condition 3.2.1, Condition 3.2.2): Is an equilibrium exactly when $F_A^{-1}(\frac{1}{2}) < \frac{\theta_A+\theta_B}{2} < F_B^{-1}(\frac{1}{2})$.

(3) (Condition 3.2.1, Condition 3.2.6): Requires that $\theta_A = \rho_A = F_B^{-1}(\frac{1}{2})$.

(4) (Condition 3.2.1, Condition 3.2.7): Requires that $\theta_A = \rho_A > F_B^{-1}(\frac{1}{2})$.

(5) (Condition 3.2.1, Condition 3.2.8): Requires that $\theta_B \leq \rho_A = \theta_A$.

(6) (Condition 3.2.2, Condition 3.2.1): Requires that $\theta_A = \rho_A > \rho_B = \theta_B$.

(7) (Condition 3.2.2, Conditions 3.2.2 and 3.2.6 to 3.2.9): Requires contradictory orderings of ρ_A and ρ_B.

(8) (Condition 3.2.2, Condition 3.2.3): Requires that $\theta_A = \rho_A = F_B^{-1}(\frac{1}{2})$.

(9) (Condition 3.2.2, Condition 3.2.4): Requires that $\theta_A = \rho_A > F_B^{-1}(\frac{1}{2})$.

(10) (Condition 3.2.2, Condition 3.2.5): Requires that $\theta_B \leq \rho_A = \theta_A$.

(11) (Condition 3.2.3, Conditions 3.2.1, 3.2.3 to 3.2.5 and 3.2.9): Requires contradictory orderings of ρ_A and ρ_B.

(12) (Condition 3.2.3, Condition 3.2.2): Requires that $\theta_B = \rho_B = F_A^{-1}(\frac{1}{2})$.

(13) (Condition 3.2.3, Condition 3.2.6): Can be equilibrium when $\rho_B = F_A^{-1}(\frac{1}{2})$ and $\rho_A = F_B^{-1}(\frac{1}{2})$. Note this implies the condition $F_B^{-1}(\frac{1}{2}) < \frac{\rho_A+\rho_B}{2} < F_A^{-1}(\frac{1}{2})$ exactly when $F_B^{-1}(\frac{1}{2}) < F_A^{-1}(\frac{1}{2})$.

(14) (Condition 3.2.3, Condition 3.2.7): Can be equilibrium when: $\rho_B = F_A^{-1}(\frac{1}{2})$ and $\theta_B \geq \rho_A > F_B^{-1}(\frac{1}{2})$. And $F_B^{-1}(\frac{1}{2}) < \frac{\rho_A+\rho_B}{2} < F_A^{-1}(\frac{1}{2})$ and $\rho_A < \rho_B = F_A^{-1}(\frac{1}{2})$. For example, $F_B^{-1}(\frac{1}{2}) = 0.1$, $\rho_B = F_A^{-1}(\frac{1}{2}) = 0.9$, $\theta_B = 0.95$, with $\rho_A = 0.15$.

(15) (Condition 3.2.3, Condition 3.2.8): Requires that $\theta_B \leq \rho_A < F_B^{-1}(\frac{1}{2})$.

(16) (Condition 3.2.4,-): Requires that $\theta_A \geq \rho_B > F_A^{-1}(\frac{1}{2})$.

(17) (Condition 3.2.5, Conditions 3.2.1, 3.2.3 to 3.2.5 and 3.2.9): Requires contradictory orderings of ρ_A and ρ_B.

(18) (Condition 3.2.5, Condition 3.2.2): Requires that $\theta_B = \rho_B < F_A^{-1}(\frac{1}{2})$.

(19) (Condition 3.2.5, Condition 3.2.6): Can be equilibrium when: $\theta_A \leq \rho_B < F_A^{-1}(\frac{1}{2})$. And $\rho_A = F_B^{-1}(\frac{1}{2})$. And $\rho_A < \rho_B$. And $F_B^{-1}(\frac{1}{2}) < \frac{\rho_A+\rho_B}{2} < F_A^{-1}(\frac{1}{2})$. For example, $\theta_A = 0.1$, $F_A^{-1}(\frac{1}{2}) = 0.9$, $F_B^{-1}(\frac{1}{2}) = 0.15$, with $\rho_A = F_B^{-1}(\frac{1}{2})$ and $\rho_B = 0.5$.

(20) (Condition 3.2.5, Condition 3.2.7): Can be equilibrium when: $\theta_A \leq \rho_B < F_A^{-1}(\frac{1}{2})$. And $\theta_B \geq \rho_A > F_B^{-1}(\frac{1}{2})$. And $\rho_A < \rho_B$. And $F_B^{-1}(\frac{1}{2}) < \frac{\rho_A+\rho_B}{2} < F_A^{-1}(\frac{1}{2})$. For example, $\theta_A = 0.1$, $\theta_B = 0.9$, $F_A^{-1}(\frac{1}{2}) = 0.9$, $F_B^{-1}(\frac{1}{2}) = 0.15$, with $\rho_A = 0.16$ and $\rho_B = 0.5$.

(21) (Condition 3.2.5, Condition 3.2.8): Requires that $\theta_B \leq \rho_A < F_B^{-1}(\frac{1}{2})$.

(22) (Condition 3.2.6, Condition 3.2.1): Requires that $F_A^{-1}(\frac{1}{2}) = \rho_B = \theta_B$.

(23) (Condition 3.2.6, Conditions 3.2.2 and 3.2.6 to 3.2.9): Requires contradictory orderings of ρ_A and ρ_B.

(24) (Condition 3.2.6, Condition 3.2.3): Can be equilibrium when $\rho_A = F_B^{-1}(\frac{1}{2})$ and $\rho_B = F_A^{-1}(\frac{1}{2})$. Note this implies the condition $F_A^{-1}(\frac{1}{2}) < \frac{\rho_A+\rho_B}{2} < F_B^{-1}(\frac{1}{2})$ exactly when $F_A^{-1}(\frac{1}{2}) < F_B^{-1}(\frac{1}{2})$.

(25) (Condition 3.2.6, Condition 3.2.4): Is an equilibrium exactly when $\rho_B = F_A^{-1}(\frac{1}{2}) < \frac{\rho_A+\rho_B}{2} < F_B^{-1}(\frac{1}{2}) < \rho_A \leq \theta_B$. For example, $F_A^{-1}(\frac{1}{2}) = 0.15$, $F_B^{-1}(\frac{1}{2}) = 0.5$, $\theta_B = 0.9$, and $\rho_A = 0.55$ and $\rho_B = 0.15$.

(26) (Condition 3.2.6, Condition 3.2.5): Requires that $\theta_B \leq \rho_A < F_B^{-1}(\frac{1}{2})$.

(27) (Condition 3.2.7,-): Requires that $\theta_A \geq \rho_B > F_A^{-1}(\frac{1}{2})$.

(28) (Condition 3.2.8, Condition 3.2.1): Requires that $\theta_B = \rho_B < F_A^{-1}(\frac{1}{2})$.

(29) (Condition 3.2.8, Conditions 3.2.2 and 3.2.6 to 3.2.9): Requires contradictory orderings of ρ_A and ρ_B.

(30) (Condition 3.2.8, Condition 3.2.3): Is an equilibrium exactly when $\theta_A \leq \rho_B < F_A^{-1}(\frac{1}{2}) < \frac{\rho_A+\rho_B}{2} < F_B^{-1}(\frac{1}{2}) = \rho_A$. For example, $\theta_A = 0.1$, $F_A^{-1}(\frac{1}{2}) = 0.15$, $F_B^{-1}(\frac{1}{2}) = 0.5$, and $\rho_A = 0.5$ and $\rho_B = 0.1$.

(31) (Condition 3.2.8, Condition 3.2.4): Is an equilibrium exactly when $\theta_A \leq \rho_B < F_A^{-1}(\frac{1}{2}) < \frac{\rho_A+\rho_B}{2} < F_B^{-1}(\frac{1}{2}) < \rho_A \leq \theta_B$. For example, $\theta_A = 0.1$, $\theta_B = 0.7$, $F_A^{-1}(\frac{1}{2}) = 0.15$, $F_B^{-1}(\frac{1}{2}) = 0.5$, and $\rho_A = 0.6$ and $\rho_B = 0.1$.

(32) (Condition 3.2.8, Condition 3.2.5): Requires that $\theta_B \leq \rho_A < F_B^{-1}(\frac{1}{2})$.

(33) (Condition 3.2.9, Conditions 3.2.1 to 3.2.8): Requires contradictory orderings of ρ_A and ρ_B.

(34) (Condition 3.2.9, Condition 3.2.9): Let $\rho = \rho_A = \rho_B$. If $F_A^{-1}(\frac{1}{2}) < F_B^{-1}(\frac{1}{2})$, either $\rho < F_B^{-1}(\frac{1}{2})$ or $\rho > F_A^{-1}(\frac{1}{2})$. In the former case, it would follow that $\rho < F_B^{-1}(\frac{1}{2}) < \theta_B$. In the latter case, it would follow that $\rho > F_A^{-1}(\frac{1}{2}) > \theta_A$. Both use the assumed ordering on an individual candidate's beliefs and ideal policy. This arrangement is not allowed by Condition 3.2.9. Thus, this cannot be equilibrium. Alternatively, consider

the case $F_B^{-1}(\frac{1}{2}) < F_A^{-1}(\frac{1}{2})$. If $\rho < F_B^{-1}(\frac{1}{2})$ or $\rho > F_A^{-1}(\frac{1}{2})$, the same arguments as above would continue to apply. However, if $F_B^{-1}(\frac{1}{2}) < \rho < F_A^{-1}(\frac{1}{2})$, this would be an equilibrium exactly when $\theta_B \geq \rho$ and $\theta_A \leq \rho$. Note that this is implied given that $\theta_A \leq F_B^{-1}(\frac{1}{2}) \leq F_A^{-1}(\frac{1}{2}) \leq \theta_B$. If $\rho = F_B^{-1}(\frac{1}{2})$ this would be an equilibrium exactly when $\theta_A \leq \rho$; if $\rho = F_A^{-1}(\frac{1}{2})$ this would be an equilibrium exactly when $\theta_B \geq \rho$. Note again this is implied given that $\theta_A \leq F_B^{-1}(\frac{1}{2}) \leq F_A^{-1}(\frac{1}{2}) \leq \theta_B$.

□

Proof of Corollary 3.5. Equilibrium 3.5.1 is an equilibrium by Condition 3.2.9 for $i = A$ and Condition 3.2.9 for $i = B$, using the discussion of this case in the proof of Corollary 3.4.

Equilibrium 3.5.2 encompasses multiple cases depending on whether the inequalities involving ρ are equalities or strict inequalities:

(1) If $\rho_A = F_B^{-1}(\frac{1}{2}) < \frac{\rho_A+\rho_B}{2} < F_A^{-1}(\frac{1}{2}) = \rho_B$, this is an equilibrium by Condition 3.2.3 for $i = A$ and Condition 3.2.6 for $i = B$.

(2) If $\rho_A = F_B^{-1}(\frac{1}{2})$ and $\theta_A \leq \rho_B < F_A^{-1}(\frac{1}{2})$, this is an equilibrium by Condition 3.2.5 for $i = A$ and Condition 3.2.6 for $i = B$.

(3) If $F_B^{-1}(\frac{1}{2}) < \rho_A \leq \theta_B$ and $F_A^{-1}(\frac{1}{2}) = \rho_B$, this is an equilibrium by Condition 3.2.3 for $i = A$ and Condition 3.2.7 for $i = B$.

(4) If $F_B^{-1}(\frac{1}{2}) < \rho_A \leq \theta_B$ and $\theta_A \leq \rho_B < F_A^{-1}(\frac{1}{2})$, this is an equilibrium by Condition 3.2.5 for $i = A$ and Condition 3.2.7 for $i = B$.

There are no other pure strategy equilibria by the same arguments as in Corollary 3.4. Indeed, the arguments in the proof of that part of Corollary 3.4 are written so that they also cover this case. □

Proof of Corollary 3.6. Consider $\rho_A \neq \rho_B$.

Consider the case $|\rho_i - F^{-1}(\frac{1}{2})| < |\rho_j - F^{-1}(\frac{1}{2})|$, so candidate i wins the election. By Lemma 3.1.1, for this to be an equilibrium, it must be that $\rho_i = \theta_i$. By Lemma 3.1.2, for this to be an equilibrium, it must be that $\theta_i = \rho_i = F^{-1}(\frac{1}{2})$ or $\theta_j \geq \rho_i = \theta_i > F^{-1}(\frac{1}{2})$ or

$\theta_j \leq \rho_i = \theta_i < F^{-1}(\frac{1}{2})$. This would require an arrangement of candidates' ideal policies that is ruled out by assumption.

Alternatively, consider the case $|\rho_i - F^{-1}(\frac{1}{2})| = |\rho_j - F^{-1}(\frac{1}{2})|$. From Lemma 3.1.3, it cannot be an equilibrium for candidates to announce different policies but tie the election.

Therefore, $\rho_A \neq \rho_B$ is inconsistent with equilibrium.

Since a pure strategy equilibrium exists by Theorem 3.2, the equilibrium exists with $\rho_A = \rho_B = \rho$. Suppose that $F^{-1}(\frac{1}{2}) < \rho$. By Lemma 3.1.4, for this to be an equilibrium, it must be that $\theta_i \geq \rho$ for both $i \in \{A, B\}$. However, that is false, as $\theta_A < F^{-1}(\frac{1}{2})$. Therefore, such ρ cannot be an equilibrium. Similarly, ρ such that $F^{-1}(\frac{1}{2}) > \rho$ cannot be an equilibrium.

Therefore, the equilibrium must be $\rho_A = \rho_B = F^{-1}(\frac{1}{2})$. And, indeed, this is an equilibrium by Condition 3.2.9 for both $i = A$ and $i = B$. □